بسم الله الرحمن الرحيم

Dr. Lafta Alqaisy

Numerical Analysis

التحليلات العدديه

A set of equations having the form

لنأخذ المجموعة التالية من المعادلات الخطية

$$a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + \cdots a_{1n}X_n = y_1$$
$$a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + \cdots a_{2n}X_n = y_2$$
$$a_{31}X_1 + a_{32}X_2 + a_{33}X_3 + \cdots a_{3n}X_n = y_3$$
$$\vdots$$
$$a_{m1}X_1 + a_{m2}X_2 + a_{m3}X_3 + \cdots + a_{mn}X_n = y_3$$

يمكن اعادة كتابة هذه المعادلات على شكل مصفوفة

we can rearrangment this equations as matrix

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & & & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_n \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_m \end{pmatrix} \Rightarrow [A]\underline{X} = \underline{Y}$$

يمكن حل هذه المعادلات بطرق عدديه عديده

we can solve this set equation by many method

١- الطرق المباشرة

Direct Method

Can be solve using one of the following case

يمكن حلها

(i) [A] is reduced to [D] diagonal matrix

(أ) يمكن جعل المصفوفة [A] مصفوفة قطرية بعد سلسلة عمليات متكررة

$$\begin{pmatrix} d_{11}X_1 & 0 & 0 - 0 \\ 0 & d_{22}X_2 & 0 \\ 0 & 0 & d_{33}X_3 \\ & & & d_{mn} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_m \end{pmatrix} = \begin{pmatrix} C_1 \\ C_2 \\ \vdots \\ C_m \end{pmatrix}$$

والحل يكون

$$X_i = \frac{C_i}{d_{ii}} \quad \text{by assume } d_{ii} \neq 0$$

(ب)

(ii) [A] is reduced to lower triangular matrix of order $(n*n)$ [L]

$$\begin{bmatrix} l_{11} & 0 & 0 & \cdots & 0 \\ l_{21} & l_{22} & 0 & \cdots & 0 \\ l_{31} & l_{32} & l_{33} & \cdots & 0 \\ \vdots & \vdots & & \ddots & \\ l_{n1} & l_{n2} & \cdots & \cdots & l_{nn} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_n \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ \vdots \\ C_n \end{bmatrix}$$

forward substitution method ↓

Then لذلك

$$X_1 = \frac{C_1}{l_{11}}$$

$$X_2 = (C_2 - l_{21} X_1)/l_{22}$$

$$X_3 = (C_3 - l_{31} X_1 - l_{32} X_2)/l_{33}$$

$$X_n = \left(C_n - \sum_{J=1}^{n-1} l_{nJ} X_J\right)/l_{nn}$$

$$\boxed{X_i = \left(C_i - \sum_{J=1}^{i-1} l_{iJ} X_J\right)/l_{ii}}$$

(iii) [A] is reduced to upper tringular matrix of order $n*n$ [U]

(ح) يمكن تقليل المصفوفة المذكورة الى مصفوفة المثلث العلوي برتبة $(n*n)$

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ 0 & u_{22} & u_{23} & \cdots & u_{2n} \\ 0 & 0 & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & & u_{nn} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_n \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ \vdots \\ C_n \end{bmatrix}$$

backward substitution method ↑

The solution is carried out using the backword substitution method

$$X_n = C_n/u_{nn}$$

$$X_{n-1} = (C_{n-1}) - u_{n-1,n} X_n)/u_{n-1,n-1}$$

$$\vdots$$

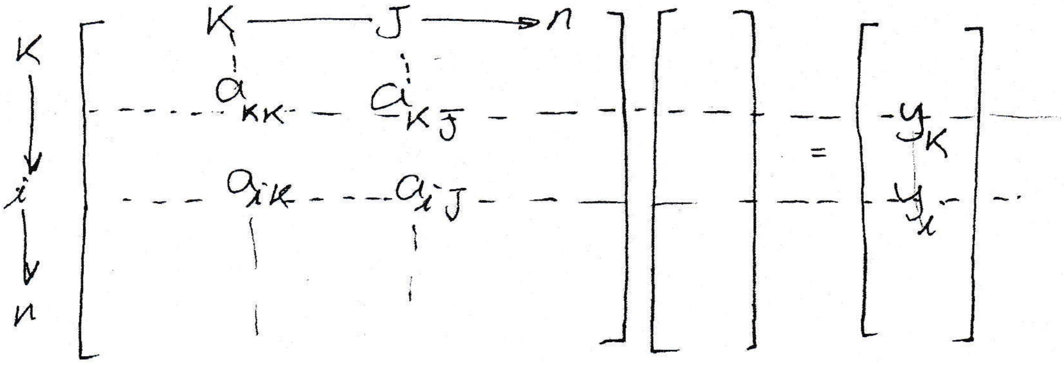
$$X_i = \left(C_i - \sum_{J=i+1}^{n} u_{iJ} X_J\right)/u_{ii}$$

$$X_1 = \left(C_1 - \sum_{J=2}^{n} u_{iJ} X_J\right)/u_{11}$$

Gauss Elimination method

[A] is reduced to an upper (or Lower) triangular matrix then it can be solved the resulting system by back or forward substitution

يمكن تقليله الى مصفوفة عليا او سفلى ومن ثم حلها بطريقه الحل من الجهه السفلى او الحل من الجهه العليا



to mak $a_{iK} = 0$ لجعل $a_{iK} = 0$

i) divide the K^{th} equation by a_{KK} (provided $a_{KK} \neq 0$)

ii) multiply the resulting equation by $(-a_{iK})$

iii) add the result to i^{th} equation

أ- نقسم الصف على a_{KK}

ب- نضرب الصف ب $(-a_{iK})$

ج- نضيف الناتج الى المعادلة الجديدة

$$a_{iJ_{new}} = a_{iJ_{old}} - a_{iK}\frac{a_{KJ}}{a_{KK}}$$

$$y_{i_{new}} = y_{i_{old}} - a_{iK}\frac{y_K}{a_{KK}}$$

EX solve this set of equation by using Gauss Elimation method

حل المعادلات التالية باستخدام طريقة (كاوس ايمنيشن)

$$3X_1 - X_2 + 2X_3 = 12 \quad \text{---- (1)}$$
$$X_1 + 2X_2 + 3X_3 = 11 \quad \text{---- (2)}$$
$$2X_1 - 2X_2 - X_3 = 2 \quad \text{---- (3)}$$

Step 1 $C_1 = \frac{a_{21}}{a_{11}}$, $C_2 = \frac{a_{31}}{a_{11}}$ المرحلة الاولى

التخلص من X_1 في المعادلة (2)

$i * \frac{a_{KJ}}{a_{KK}} = 1 + \frac{-1*3}{3} = 0$, $= 2 - \frac{1*(-1)}{3} = \frac{7}{3}$, $= 3 - \frac{2*1}{3} = \frac{7}{3}$, $11 - \frac{12}{3} = \frac{33-12}{3}$

$$3X_1 - X_2 + 2X_3 = 12 \quad \text{------ (4)}$$
$$7X_2 + 7X_3 = 21 \quad \text{---- (5)} \quad -2 + \frac{2}{3}*1 = \frac{-6+2}{3} = \frac{-4}{3}$$
$$-4X_2 - 7X_3 = -18 \quad \text{------ (6)} \quad -1 - 2*\frac{2}{3} = -\frac{7}{3}$$

Step 2 $C = \frac{a_{32}}{a_{22}}$

$$\therefore a_{33} = a_{33} - a_{23} * \frac{a_{32}}{a_{22}}$$

$$a_{33} = -7 \neq 7 * \frac{-4}{7} = \frac{21}{7}$$

$$\therefore \quad 3X_1 - X_2 + 2X_3 = 12 \quad \text{———} (7)$$

$$7X_2 + 7X_3 = 21 \quad \text{———} (8)$$

$$-21X_3 = -42 \quad \text{———} (9)$$

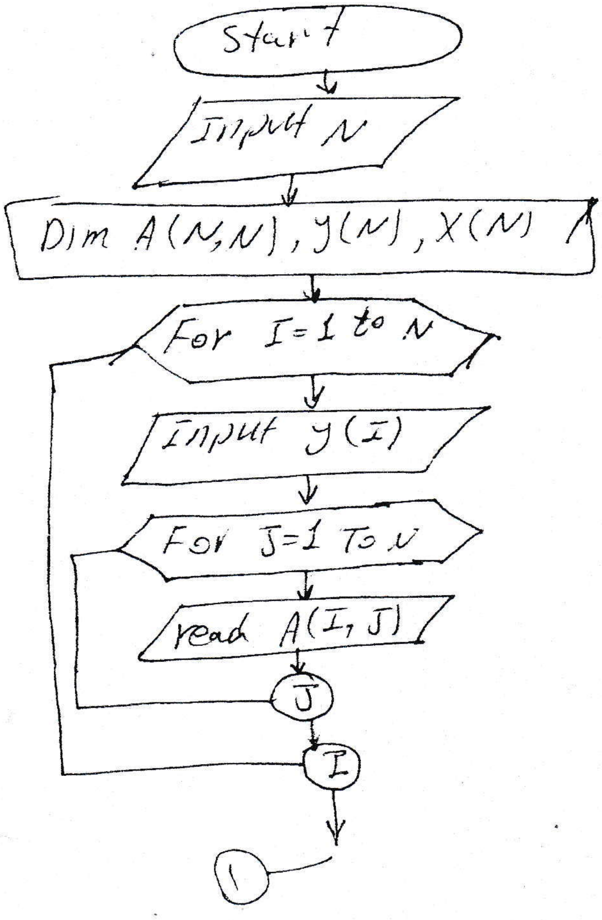
from (9) we find $X_3 = 2$ and from (8) $X_2 = 1$ and from (7) $X_1 = 3$

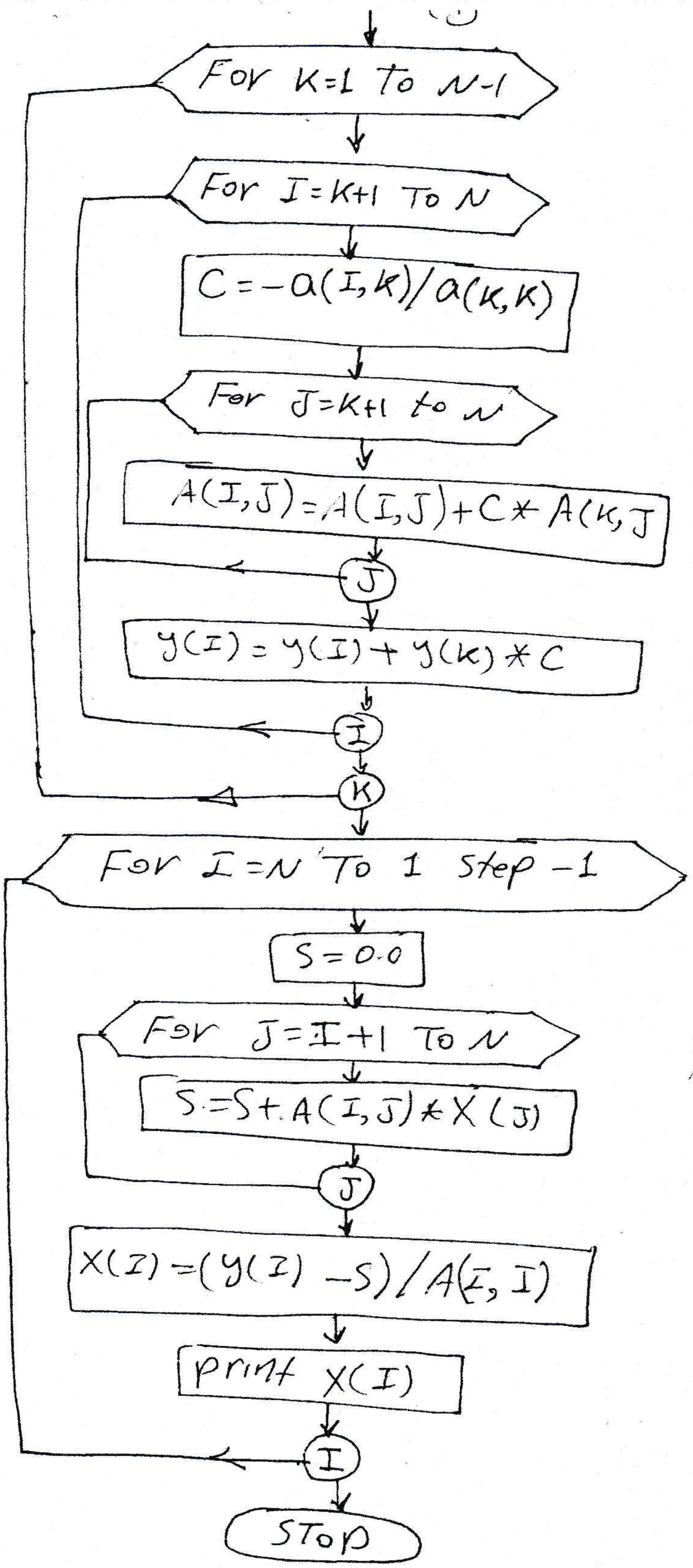
H.W find the solution of set of equation by using Gauss elemination method

جد حل لمجموعه المعادلات باستخدام طريقة Gauss elemenation

$$4.44X - 9.94y + 2.41Z = 5.36$$
$$8.24X - 2.02y - 2.18Z = 9.34$$
$$0.93X + 3.86y + 11.66Z = 2.57$$

وباستخدام البرمجه في حاله وجود عدد كبير من المعادلات يمكن كتابه برنامج الخاص باستخدام Gauss elemenation ← back word





$$X_i = \frac{\left(C_i - \sum_{J=i+1}^{n} u_{iJ} X_J\right)}{u_{ii}}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{1n} \\ 0 & a_{22} & a_{23} & a_{2n} \\ 0 & 0 & a_{33} & \\ 0 & 0 & 0 & \end{bmatrix}$$

ملاحظة من مساوئ هذه الطريقة (Gauss elemination) هي عدم امكانية الحل في حالة كون $a_{11}=0$ وفي هذه الحالة يجب اعادة ترتيب المعادلات لتجنب حدوث ذالك

The Gauss elemination method ~~was~~ fills If $a_{11}=0$. That must be rearrengment.

طريقة التحليل المثلثي (LU)

Triangular decomposition (LU decomposition)

ملاحظات

Notes

① المصفوفة المربعة A يمكن تحليلها الى مصفوفتين مثلثتين المصفوفة الاولى مثلثية سفلى (L) والثانية مصفوفة مثلثية علوية (U)

$A = LU$

we can devided the squr matrix in to two matrix upper triangular matrix and Lower tringular matrix $A = L \cdot U$

② للحصول على المصفوفتين U,L نتبع الخطوات التالية نفرض المصفوفة A محايا

) to get two matrix U,L we must do this proseger

Let matrix A

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

نأخذ اولاً العناصر القطرية للمصفوفة L تساوي 1 عليه فان

Let the diagonal to L matrix equal 1

$$L = \begin{pmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{pmatrix}$$

كذلك

and

$$U = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{pmatrix}$$

using the equation $LU = A$, $LU = A$ نستخدم العلاقة

$$\begin{pmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{pmatrix} \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{33} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$\therefore$ $\boxed{U_{11} = a_{11}}$ $\quad L_{21} U_{11} = a_{21} \Rightarrow \boxed{L_{21} = \frac{a_{21}}{U_{11}}}$

$\boxed{U_{12} = a_{12}}$ $\quad L_{21} U_{12} + U_{22} = a_{22} \Rightarrow \boxed{U_{22} = a_{22} - L_{21} U_{12}}$

$\boxed{U_{13} = a_{13}}$

$$L_{21} U_{13} + U_{23} = a_{23}$$

من الافضل ان يكون a_{11} اكبر عنصر في العمود الاول

That best do a_{11} larger number in colum one

$$L_{31}U_{11} = a_{31} \Rightarrow L_{31} = \frac{a_{31}}{U_{11}}$$

$$L_{31}U_{12} + L_{32}U_{22} = a_{32}$$

$$L_{32}U_{22} = a_{32} - L_{31}U_{12}$$

$$L_{32} = \frac{a_{31} - L_{31}U_{12}}{U_{22}}$$

$$L_{31}U_{13} + L_{32}U_{23} + U_{33} = a_{33}$$

$$U_{33} = a_{33} - L_{31}U_{13} - L_{32}U_{23}$$

حل منظومة معادلات خطية بطريقة التحليل التالي

Solve the system linear matrix

$$a_{11}X_1 + a_{12}X_2 + a_{13}X_3 = C_1$$
$$a_{21}X_1 + a_{22}X_2 + a_{23}X_3 = C_2$$
$$a_{31}X_1 + a_{32}X_2 + a_{33}X_3 = C_3$$

بطريقة المصفوفات يمكن كتابة المنظومة اعلاه بالشكل التالي

$$A\underline{X} = C$$

we can write the set equation as matrix $A\underline{X} = C$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix}$$

المصفوفة A يمكن تحليلها الى مصفوفتين U, L عليه نحصل على الاتي

$$LU\underline{X} = \underline{C}$$

the matrix (A) can anlaysis in to two matrix U, L

$$LU\underline{X} = \underline{C}$$

Let $U\underline{X} = y$ ---- (11) ...... نفترض ان

from (11) get $Ly = C$ ---(12) ومنه نحصل على

from (12) get ومن المعادلة 12 نحصل

$$\begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix}$$

$y_1 = c_1$

$L_{21} y_1 + y_2 = c_2$

$L_{31} y_1 + L_{32} y_2 + y_3 = c_3$

نعوض عن قيم y_1 , y_2 , y_3 في المعادله (11)

$$\begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$u_{11} x_1 + u_{12} x_2 + u_{13} x_3 = y_1$$
$$u_{22} x_2 + u_{23} x_3 = y_2$$
$$u_{33} x_3 = y_3$$

ومنه نحصل على قيم x_1, x_2, x_3

from it get x_1, x_2, x_3 ....

مثال استخدام طريقه تحليل المثلثي لحل المنظومه التاليه مقربا حسابتك الى ثلاث مراتب عشريه (3D)

EX using Trangular anlatical to solve the set ~~matrix~~ equation approximat to 3D

$$8x + 3y - z = 2$$
$$2x - 6y - 2z = 5$$
$$x + y + 4z = 4$$

$$\underset{L}{\begin{pmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{pmatrix}} \underset{U}{\begin{pmatrix} 8 & 3 & -1 \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}} = \underset{A}{\begin{pmatrix} 8 & 3 & -1 \\ 2 & -6 & -2 \\ 1 & 1 & 4 \end{pmatrix}}$$

$$\begin{pmatrix} 8 & 3 & -1 \\ 8L_{21} & 3L_{21} + u_{22} & -L_{21} + u_{23} \\ 8L_{31} & 3L_{31} + L_{32}u_{23} & -L_{31} + L_{32}u_{23} + u_{33} \end{pmatrix} = \begin{pmatrix} 8 & 3 & 1 \\ 2 & -6 & -2 \\ 1 & 1 & 4 \end{pmatrix}$$

$L_{21} = 0.25$ $\quad u_{22} = -6.75$

$L_{31} = 0.125$ $\quad u_{23} = -1.75$

$L_{32} = -0.093$ $\quad u_{33} = 3.963$

Let $LW = C$
$UX = W$

نفرض $LW = C$
$UX = W$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0.25 & 1 & 0 \\ 0.125 & -0.093 & 1 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ 4 \end{pmatrix}$$

$w_1 = 2$; $w_2 = 4.5$; $w_3 = 4.169$

$$\begin{pmatrix} 8 & 3 & -1 \\ 0 & -6.75 & -1.75 \\ 0 & 0 & 3.963 \end{pmatrix} \begin{pmatrix} X \\ y \\ Z \end{pmatrix} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$$

$Z = 1.052$ $y = -0.939$ $X = 0.734$

عند التحقق $U.L = A$

H.W solve the set equation by using trangular analysis

$4.44X - 9.94y + 2.41Z = 5.36$
$8.24X - 2.02y - 2.18Z = 9.34$
$0.93X + 3.86y + 11.66Z = 2.57$

Numerical Solution of non Linear equation

عندما تكون المعادله بسيطه مثل $X^2-4X-3=0$ يمكن استخدام طريقه الدستور لحلها ولكن عندما تكون المعادله $2X^4+X^3-3X^2+4X-1=0$ او $X = 2\sin X$ فلا يوجد هناك معادله او طريقه معينه لحلها ولذلك يتم استخدام الطرق التقريبية

The following equation are nonlinear algebric Equation

① $\cos X - X = 0$ ② $X + \ln X = 0$ ③ $X^3 + 2X - 3\sin X = 0$

Can be solved by the following method

1) Iteration method

١) طريقه التكرار البسيط

تعتبر هذه الطريقة سهله ويمكن تطويعها لمختلف المسائل فاذا كانت المعادله

$f(X) = 0 \Rightarrow X = F(X)$

If the equation whose roots we are attempting to find is $f(X) = 0$ then rewrite the eq. in the form
$X = f(X)$

$X_1 = f(X_0)$

$X_2 = f(X_1)$

$X_3 = f(X_2)$

$$\boxed{X_{n+1} = F(X_n)}$$

we stop the Calculation If

$$|X_{n+1} - X_n| \leq \epsilon$$

where ϵ = accuracy

يمكن التوقف عن التكرار عندما يكون الفارق بين القيم القديمة والجديدة اقل من قيمة الدقة المطلوبة

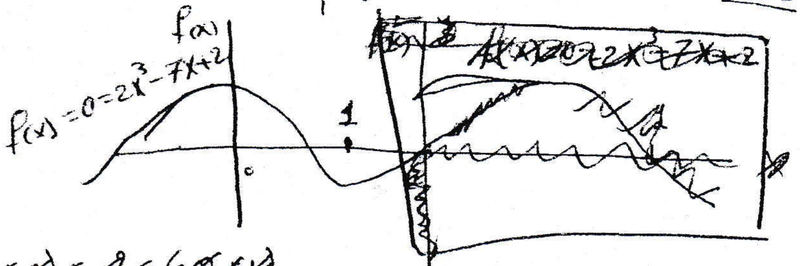
EX solve the fallawing non Linear algebric Equation to find one real non-Zero roots with $\epsilon = 0.001 = 10^{-3}$

$f(X) = 2X^3 - 7X + 2 = 0$

مثال حل المعادلة غير الخطية لايجاد التالية

Solution $X_{n+1} = f(X_n)$

$$X_{n+1} = \frac{2}{7}(X_n^3 + 1)$$



يمكن رسم المعادلة على احداثي (X, Y) ولقيم تقريبية لـ (X) ومن ثم نحاول ايجاد جذر المعادلة الموجود بين قيم X=0 و X=1 وذلك اعتمادا على الجدول التالي رقم (1)

there is root

X	−1	0	1
f(x)	+Ve	+Ve	−Ve

الجدول رقم (1)

ملاحظة * في حالة انقلاب قيمة الناتجة من المعادلة بعد التعويض في المعادلة الاصلية وبعد الانقلاب من الموجب الى السالب فتكون قيمة الجذر المطلوب محصورة بين آخر قيمة موجبة وآخر قيمة سالبة

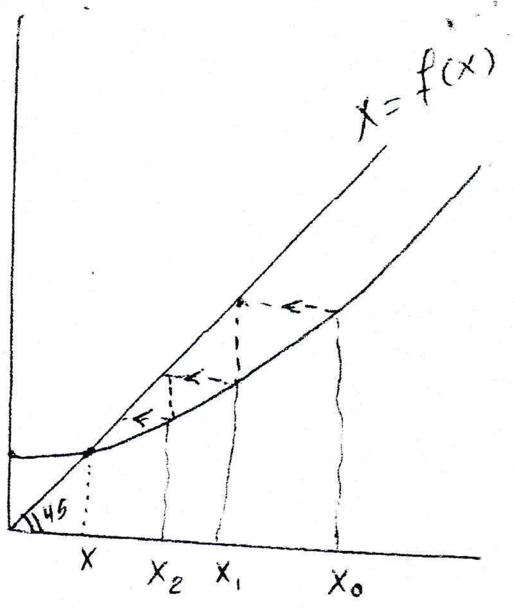
Let $X_0 = 1 \Rightarrow X_0 = \frac{2}{7}(1+1) = 0.571$

n	X	X_{n+1}
0	1	0.571
1	0.571	0.339
2	0.339	0.297
3	0.297	0.293
4	0.293	0.293

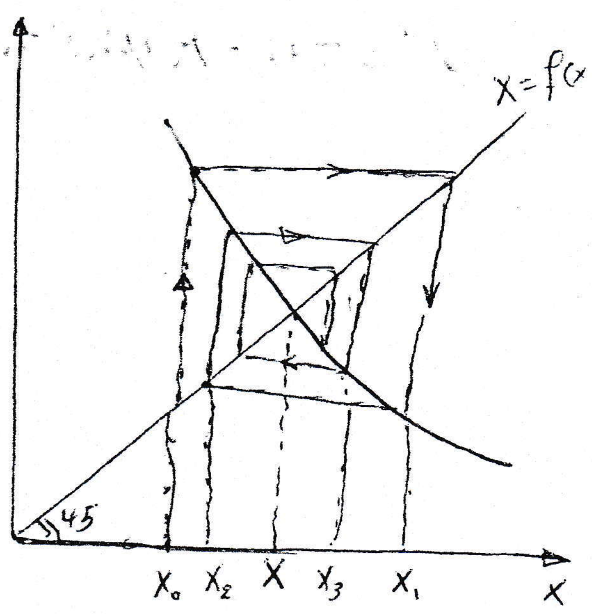
(الجدول رقم 2)

This method is not always possible to find a solution due to the problem of using the simple iterative method due to the problem of convergence. Examples for illustlating the convergence and divergence of simple Itertion method is

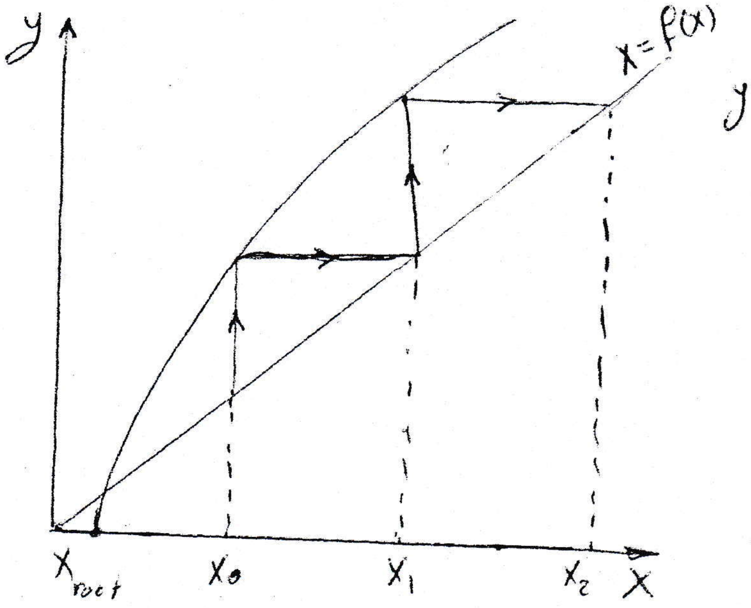
هذه الطريقة ليست ناجحة دائما لايجاد حل الا في حالة كون الدالة متقاربة اي باستمرار التكرار تتقارب من بعضها القيم X_n , X_{n+1} اما في حالة التباعد فلا يمكن حل المعادلة بهذه الطريقة



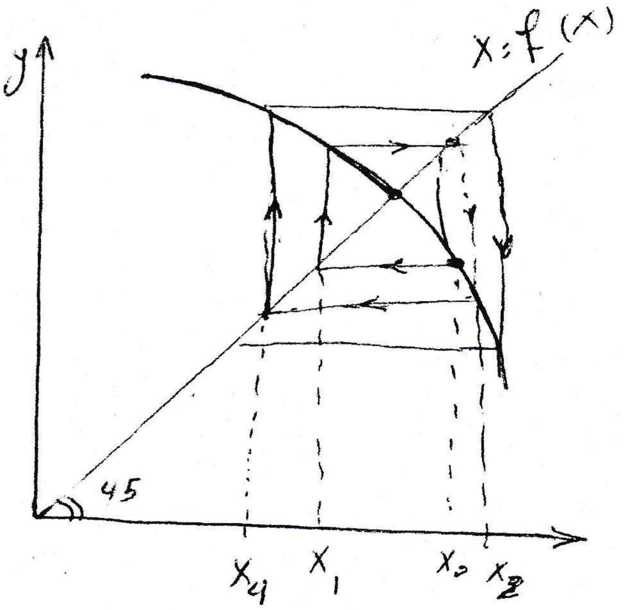
Converging process



Converging process



Diverging process



Diverging process

H.W solve the following non linear algebric Equation by Iteration method with $\epsilon = 0.0001$

واجب حل المعادلات الغير خطيه التاليه بطريقه التكرار

① $x + \ln x = 2$

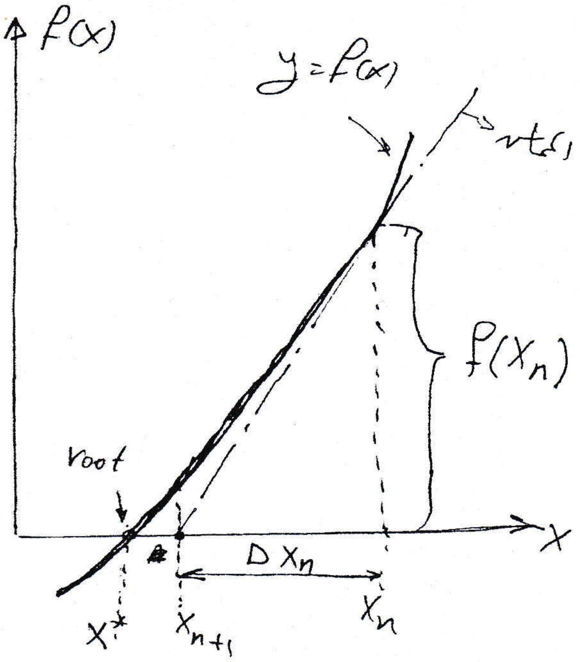
② $\tan x - x = 0$

③ $x^2 - \sin x = 0$

Newtan- Raphson Method

تعتبر هذه الطريقة جيده للاسراع في عملية التقارب convergence للمعادلات الخطية فاذا كانت عملية التكرار قد وصلت التقريب عند النقطة X_n اذن يراد زيادة صغيره مقدارها DX_n للوصول الى الحل او الجذر X^* كما في الشكل وبعد وضع Tayler Series بالشكل التالي

If the iteration has reached the point X_n then an increment DX_n is reguired which will take the process to solution point X^* as shown



$$f(X^*)=0=f(X_n)+DX_n f'(X_n)+\frac{DX_n^2}{2!}f''(X_n)+\cdots$$

where $DX_n \to$ very small neglect the 2^{nd}

$$f(X_n)+DX_n f'(X_n)=f(X_1)=0$$

Taking the first two term r-h-s (right hand said)

$$slop=f'(X)=\frac{f(X_n)}{DX_n}$$

$$\therefore DX_n=\frac{f(X_n)}{f'(X)}$$

But $X_{n+1}=X_n-DX_n$

$$\therefore X_{n+1}=X_n-\frac{f(X_n)}{f'(X)}$$ ——— Newten rafphson method

EX solve the eq. $\frac{1}{X}+1=0$ by Newten raphson method

sol

$$X_{n+1}=X_n-\frac{f(X_n)}{f'(X_n)} \quad \text{——— } (*)$$

$$f(X_n)=\frac{1}{X_n}+1 \Rightarrow f'=\frac{-1}{X_n^2} \Rightarrow \text{at } (*)$$

$$X_{n+1}=X_n-\frac{\frac{1}{X_n}+1}{-\frac{1}{X_n^2}}=X_n+\left(\frac{1}{X_n}+1\right)*X_n^2$$

$$\therefore X_{n+1}=2X_n+X_n^2$$

at eq $\frac{1}{X}+1$

1	2	−0.5	−1			
2	1.5	−1	0			

(406)

∴ root between −0.5 ~ −1

n   X

$X_{n+1} = 2X_n + X_n^2$

$X_0 = -0.5$

$X_{n+1} = -2X_0 + X_0^2 = -2*0.5 + (-0.5)^2 = -0.75$

$\therefore X_1 = -0.75$

$X_2 = 2X_1 + X_1^2 = -0.75*2 + (-0.75)^2 = -0.837$

n	X_n	X_{n+1}
0	−0.5	−0.75
1	−0.75	−0.837
2	−0.938	−0.997
3	−0.997	−1

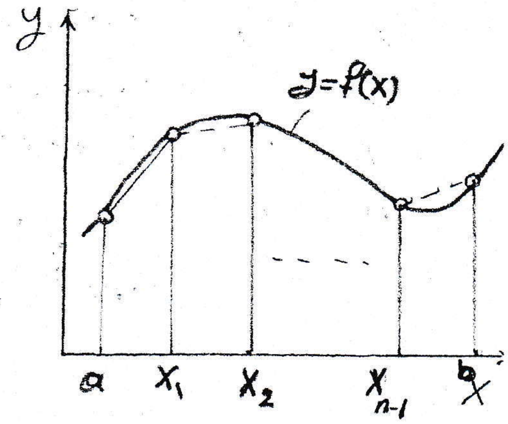
H.W
1. $2^X - 5X + 2 = 0$
2. $X^3 - 4X^2 + X - 10 = 0$
3. $\cos^4 X = 0$

Numerical Integration

Numerical integration has become a valuable tool in solving complex engineering problems. These integrals cannot always be evaluated analytically, numerical methods are usd to arrive at solution...

There are many methods in numerical methods of integral one of this :-

Trapezoidal Rule of Integration



The area under the curve of f, between a and b approximated by n trapezoids of area

$\frac{1}{2}[f(a)+f(x_1)]h$, $\frac{1}{2}[f(x_1)+f(x_2)]h \cdots$

$\frac{1}{2}[f(x_{n-1})+f(b)]h$

By taking their sum obtain the trapezoidal rule

$$J = \int_a^b f(x)\,dx \approx h\left[\frac{1}{2}f(a) + f(x_1) + f(x_2) + \cdots + f(x_{n-1}) + \frac{1}{2}f(b)\right]$$

where $h=(b-a)/n$

Ex 1 using Trapezoidal rule to solve the

$$J = \int_0^1 e^{-x^2}\,dx \qquad \text{with } n = 10$$

n	X	e^{-x^2}	
0	0	* *	1
1	0.1	0.9905	
2	0.2	0.96078	
3	0.3	0.913931	
4	0.4	0.85214	
5	0.5	0.778801	
6	0.6	0.697676	
7	0.7	0.612626	
8	0.8	0.527292	
9	0.9	0.44485	
10	1	* *	0.367879
		6.778167	1.367879

$$J = h\left[\tfrac{1}{2} f(a) + f(x_1) + f(x_2) \cdots f(x_{n-1}) + \tfrac{1}{2} f(b)\right]$$

$h = 0.1$

$$J = 0.1\left[0.5(1) + 0.9905 + 0.96078 + 0.913931 + 0.85214 + 0.778801 + 0.697676 + 0.612626 + 0.527292 + 0.44485 + 0.5 * (0.367879)\right]$$

$$= 0.746211$$

Ex 2 Find the magnitude of equation by using Trapezoidal rule

$$U = \int_0^{120} \frac{10\,dx}{30*10^3\left(-\frac{1}{3600}x^2 + \frac{1}{30}x + 1\right)} \qquad \text{If} \quad h = \text{interval} = 10$$

$$U_{AB} = \int_0^{120} \frac{dx}{-\frac{30}{36}x^2 + 100x + 3000}$$

$$\therefore f(x) = \left(-\frac{30}{36}x^2 + 100x + 3000\right)^{-1} \Rightarrow h\left[\tfrac{1}{2} f(0) + f(10) + f(20) + f(30) + f(40) + f(50) + f(60) + f(70) + f(80) + f(90) + f(100) + f(110) + \tfrac{1}{2} f(120)\right]$$

$$= 10\left[\tfrac{1}{2}(3.333) + 2.55319 + 2.14285 + 1.90476 + 1.7647 + 1.69014 + 1.6667 + 1.69014 + 1.76471 + 1.90476 + 2.1428 + 2.5531 + \tfrac{1}{2} * 3.333\right]$$

(409)

Ex for ex(1) This method (trapizoidal Rule) have an error.. The magnitude of error can be calculat for (6D)

where

$$\epsilon = \frac{(b-a)}{12} h^2 f''(\hat{t})$$ ...error eq. of Trapizoidal ru

ϵ = error estimate

$\hat{t}$ = (suitable, unknown) between a, b

Ex Estimate the error of approximate value in example (1) ---where $f = e^{-x^2}$

$f' = -2x e^{-x^2}$ ---- $f'' = 2(2x-1)e^{-x^2}$

∴ The max. and min. occur at the ends of interval. we compute((second derivitive) at two ends)

$M_2 = f''(1) = 0.735739$

$M_2^* = f''(0) = -2$

$$\therefore \epsilon_2 = \frac{(b-a)}{12} h^2 * f_2''(1) = \frac{(1-0)}{12} * 10^{-2} * 0.735739$$

$$= 0.001667$$

$$\epsilon_2^* = \frac{(b-a)h^2}{12} * f_1''(0) = \frac{(1-0)}{12} * 10^{-2} * (-2) = -0.000614$$

∴ exact value between

0.746211 − 0.000614 = 0.745597 and 0.746211 + 0.001667 = 0.7478.

∴ J = 0.746824 exact to 6D

Simpson's Rule of Integration

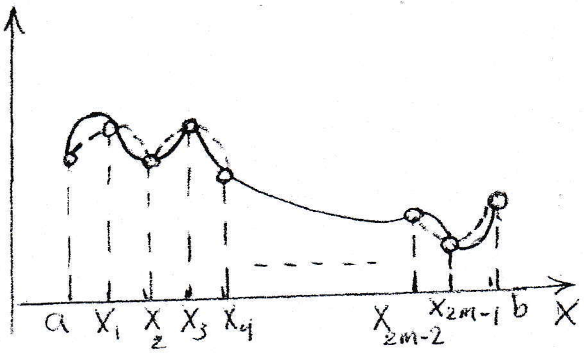
This method of integration is more accurate than Trapizoidal rule.

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left(f_0 + 4f_1 + 2f_2 + 4f_3 + \cdots + 2f_{2m-2} + 4f_{2m-1} + f_{2m}\right)$$

where $h = \frac{(b-a)}{2m}$ where ---

m = number of base points

a, b = first and End point



EX solve $\int_{u_B}^{u_A} du = \int_0^{120} \frac{10\,dx}{30 * 10^3\left(-\frac{1}{3600}x^2 + \frac{1}{30}x + 1\right)}$

where $h = 30$

sol

$$f(x) = \left(-\frac{30}{36}x^2 + 100x + 3000\right)^{-1}$$

$$f(x) = \frac{h}{3}\left[f(0) + 4f(30) + 2f(60) + 4f(90) + f(120)\right]$$

$$= 0.025238$$

Error Simpson rule

$$\epsilon = \frac{b-a}{180} h^4 f''''(\hat{t})$$ --- error eq. of simpson rule

$\therefore \hat{t}$ = (suitable, unkown) between a and b